Probability exercises - Problem Set 6 - Continuous Random Variables and Random Vectors

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For this Problem Set I added the notation **!**. It means that an exercise is more difficult than the others in the Problem Set. **!!**, as you might imagine, is translated to *only for try-harders*.

Exercise 1. *Let X be a random continuous variable with $f_X(x) = \frac{4}{3x^2}$ density function, for $1 \leq x \leq 4$. Compute the following probabilities.*

1. $P(1 < x < 3)$, $P(1 \leq X < 3)$, $P(1.5 < X < 3)$.
2. $P(X \leq 1)$, $P(X < 1)$, $P(0 \leq X \leq 2 \mid 1 \leq X \leq 3)$.
3. ***!*** $P(X \in \mathbb{N})$.
4. ***!!*** $P(X \in \mathbb{Q})$.

For the last two exercises you might want to check the concept of countable set.

Exercise 2. *Let X be a uniform random variable in $[0, 1]$. Compute the following probabilities:*

- $P(X = 1)$, $P(X = 0)$, $P(X = 0.5)$, $P(X \in [0, 1])$, $P(X \in [0, 0.5])$, $P(X \leq 0.5)$, *and* $P(X \geq 1)$
- *What is the density function of $f_X(x)$ (in the lectures), and $F_X(X)$ (you must calculate it)? Show how you can go from one to another (Show the operations).*

	X	0.3	0.5	
Y		**0**	**1**	**2**
0.6	**0**			
0.1	**1**			
	2			

Exercise 3. ***!*** *The support of a certain continuous random variable is the set* $[1,5]$. *Its density function is proportional to* x^2.

1. *Write its density function.*
2. *What is the probability of* $(1,3]$

Exercise 4. *Find the density function of the continuous random variable that has as distribution function:*

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-(1+3x)e^{-3x} & \text{if } x \geq 0 \end{cases}$$

Is this function continuous? Is it right-continuous?

Exercise 5. *Let* X *and* Y *be two discrete independent random variables with support in the set* $\{0,1,2\}$ *such that:*

$$P(X=0)=0.3, \quad P(X=1)=0.5, \quad P(Y=0)=0.6, \quad P(Y=1)=0.1$$

1. *Build a table that shows* $P(X=x, Y=y)$ *for every* (x,y) *in the common support.*
2. *Find* $P(X=Y)$.
3. *Find* $P(X+Y=2)$.

Exercise 6. *The joint density of* X *and* Y *is* $f(x,y) = \lambda^3 x e^{-\lambda(x+y)}$ *for* $x>0$ *and* $y>0$ *(0 otherwise).*

1. *Find the marginal densities and show that* X *and* Y *are independent.*
2. ***!*** *Find* $P(X \leq a, Y \leq b)$ *for every* a *and* b *positive numbers. The result might be an integral of* x *and* y *that you don't know how to calculate, just indicate the integral.*

3. *Find* $P(X \leq a)$ *for* $a > 0$.

Exercise 7. *Let* (X, Y) *a random vector with joint density function*

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}(y-x)^2}, & \text{if } -1 < x < 1, \ -\infty < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

1. ***!*** *Find* $f_X(x)$.[1]
2. *Find* $f_{Y|X}(y)$.

[1] You might want to use some integral calculator on the internet...